

Hybrid Metaheuristic Approach to the Constrained Bi-Objective Minimum Spanning TreeDana Faiq Abd^{1,2}, Haval Mohammed Sidqi³, Omed Hasan Ahmed⁴¹Department of Computer Science, College of Science, Charmo University, Sulaimani, Iraq^{2,4}Department of Information Technology, College of Science and Technology, University of Human Development, Sulaimani, Iraq³Technical College of Informatics, Sulaimani Polytechnic University, Sulaimani, Iraq**Email:** dana.fayaq@chu.edu.iq¹, dana.abd@uhd.edu.iq², haval.sidqi@spu.edu.iq³, omed.hassan@uhd.edu.iq⁴**Abstract:**

The constrained bi-objective Minimum Spanning Tree (MST) problem seeks to minimize edge weight and hop count under strict cost and delay limits. Geometry-based evolutionary algorithms, particularly AGE-MOEA, provide a distinctive advantage because they replace traditional crowding-distance survival with geometry-aware scoring based on normalized L^p distances. This mechanism explicitly models the geometric structure of the Pareto front, allowing solutions to be distributed evenly across irregular or non-convex trade-off surfaces, which enhances both diversity and convergence stability. Building on this principle, we propose a Hybrid MOCPO-AGE-MOEA that integrates the exploration strength of Multi-Objective Crested Porcupine Optimization (MOCPO) with the geometry-aware survival of AGE-MOEA. The hybrid achieves novelty through multi-level integration: alternating engines across iterations to balance exploration and exploitation, cross-injecting operators for greater adaptability, and applying feasibility-first repair to guarantee valid spanning trees underweight and hop constraints. The contributions of this study are threefold: (i) formal unification of bio-inspired exploration with geometry-based survival, (ii) a feasibility-preserving framework that ensures strict constraint satisfaction, and (iii) a balanced performance profile combining Pareto diversity, hop reduction, and competitive runtime. Experiments extend earlier benchmarks from 50-node graphs to more challenging 70-node instances, where the Hybrid consistently outperforms competitors by producing nearly three times higher Pareto diversity and the lowest hop counts, thereby confirming its scalability, robustness, and deep algorithmic strength.

Keywords: Bi-objective Minimum Spanning Tree (MST), hybrid algorithm, constrained minimum spanning tree, Network design optimization, Pareto front diversity.

الملخص:

تسعى مسألة الشجرة الممتدة الدنيا (MST) ذات الهدفين المقيدتين إلى تقليل وزن الحواف وعدد القفزات ضمن حدود صارمة للتكلفة والتأخير. توفر الخوارزميات التطورية القائمة على الهندسة، ولا سيما خوارزمية AGE-MOEA، ميزةً فريدةً لأنها تستبدل أسلوب البقاء التقليدي القائم على مسافة الازدحام بتقييم واعٍ للهندسة يعتمد على مسافات L_p المعيارية. تُنمذج هذه الآلية بشكلٍ صريح البنية الهندسية لجهة باريتو، مما يسمح بتوزيع الحلول بالتساوي عبر أسطح المقايضة غير المنتظمة أو غير المحدبة، الأمر الذي يُحسن كلاً من التنوع واستقرار التقارب. بناءً على هذا المبدأ، نقترح خوارزمية هجينة تجمع بين MOCPO و AGE-MOEA، والتي تدمج قوة الاستكشاف لخوارزمية تحسين القنفذ المتوج متعددة الأهداف (MOCPO) مع ميزة البقاء الواعية للهندسة لخوارزمية AGE-MOEA. يحقق النموذج الهجين ابتكاره من خلال التكامل متعدد المستويات: حيث يُبدل بين المحركات عبر التكرارات لتحقيق التوازن بين الاستكشاف والاستغلال، ويُدمج عوامل التشغيل لزيادة القدرة على التكيف، ويُطبق إصلاحاً قائماً على الجدوى أولاً لضمان صحة الأشجار الممتدة مع مراعاة قيود الوزن والقفز. وتتلخص إسهامات هذه الدراسة في ثلاثة جوانب: (1) التوحيد الرسمي للاستكشاف المستوحى من الطبيعة مع البقاء القائم على الهندسة، (2) إطار عمل يحافظ على الجدوى ويضمن استيفاء القيود بدقة، (3) أداء متوازن يجمع بين تنوع باريتو، وتقليل عدد القفزات، ووقت تشغيل تنافسي. وتوسع التجارب المعايير السابقة من رسوم بيانية مكونة من 50 عقدة إلى حالات أكثر تحدياً مكونة من 70 عقدة، حيث يتفوق النموذج الهجين باستمرار على منافسيه من خلال إنتاج تنوع باريتو أعلى بثلاث مرات تقريباً وأقل عدد من القفزات، مما يؤكد قابليته للتوسع، ومثابته، وقوته الخوارزمية العميقة.

الكلمات المفتاحية: شجرة الامتداد الأدنى ثنائية الهدف (MST)، الخوارزمية الهجينة، شجرة الامتداد الأدنى المقيدة، تحسين تصميم الشبكة، تنوع جبهة باريتو.

پوخته:

کیشیهی دوو نامانجی سنووردار کراوی کهمترین داره برین (MST) همولدهدات کیشی لیوار و ژمارهی هۆپ کهم بکاتهوه له ژیر سنووری توندی تیچوون و دواکهوتن. ئه‌لگۆریتمه‌کانی پهرسه‌ندن له‌سه‌ر بنه‌مای ئه‌ندازه‌یی، به‌ تایبه‌تی AGE-MOEA، سوودیکی جیاواز پێشکەش ده‌کهن چونکه‌ جیگهی مانه‌وه‌ی ته‌قلیدی-مه‌ودای قهره‌بالغی ده‌گرنه‌وه‌ به‌ خالیکردنی ئاگاداری ئه‌ندازه‌یی له‌سه‌ر بنه‌مای مه‌ودای ئاسایی L_p . ئهم مێکانیزمه‌ به‌ روونی مۆدلی پێکهاته‌ی ئه‌ندازه‌یی به‌ره‌ی پارێتو ده‌کات، رێگه‌ ده‌دات چارمه‌سه‌رمان به‌ شێوه‌یه‌کی یه‌کسان دابه‌ش بکری به‌سه‌ر رووبه‌ره‌ نارێکه‌مان یان ناچه‌قاره‌مانی مامه‌له‌کردندا، که‌ ئهمه‌ش ههم فرمه‌شه‌نی و ههم سه‌قامگیری یه‌که‌گرتن به‌رز ده‌کاته‌وه‌ له‌سه‌ر بنه‌مای ئهم بنه‌مايه‌، ئیمه‌ پێشنیاری MOCPO-AGE-MOEA ی تیکه‌ل ده‌که‌ین که‌ هیزی گه‌ران به‌ه‌وای فره‌ نامانجی باشکردنی پۆرکوپینی کریست (MOCPO) له‌گه‌ل مانه‌وه‌ی ئاگاداری ئه‌ندازه‌یی AGE-MOEA تیکه‌ل ده‌کات. هابیریدمه‌که‌ تازمه‌گه‌ری به‌ده‌ست ده‌هێنیت له‌ رێگهی یه‌که‌خشنی فره‌ ئاسته‌وه‌: گۆرینی بزوینه‌رمان له‌ سه‌رانسه‌ری دووباره‌کردنه‌وه‌ماندا بۆ هاوسه‌نگی گه‌ران و نیسته‌غاله‌کردن، ده‌رزی لێدانی ئۆپه‌راتۆره‌مان بۆ گونجانی زیاتر، و جێبه‌جێکردنی چاککردنه‌وه‌ی یه‌که‌می ئیمکانیات بۆ گه‌رمه‌نتیکردنی که‌م کیشی داره‌ په‌ستانداره‌ دروسته‌مان و سنووردارکردنی هۆپ. به‌شداریه‌یه‌کانی ئهم توێژینه‌وه‌یه‌ سێ لایه‌نه‌ (i): یه‌که‌خشنی فره‌می گه‌ران به‌ه‌وای ئیله‌امبه‌خشی بایۆلۆجی له‌گه‌ل مانه‌وه‌ی بنه‌مادار به‌ ئه‌ندازه‌یی، (ii) چوارچۆیه‌یه‌کی پاراستنی تواناکان که‌ دلنایای ده‌دات له‌ ره‌زامه‌ندی توندی سنووردارکردن، و (iii) پڕوفایلی ئه‌دای هاوسه‌نگ که‌ ههمه‌جۆریی پارێتو، که‌مه‌کردنه‌وه‌ی هۆپ و کاتی جێبه‌جێکردنی کێبڕکێ تیکه‌ل ده‌کات. تاقیکردنه‌وه‌مان پێوه‌رمانی پێشوتر له‌ گرافیکی 50 گرێه‌وه‌ درێژ ده‌که‌نه‌وه‌ بۆ نمونه‌ی 70 گرێی چالاکتر، که‌ تێیدا هابیرید به‌ به‌رده‌وامی له‌ پرکابه‌رمانی باشت‌ره‌ به‌ به‌رهمه‌نه‌نایی نزیکه‌ی سێ هینده‌ زیاتری ههمه‌جۆریی پارێتو و که‌مترین ژماره‌ی هۆپ، به‌مه‌ش قه‌بارمه‌دانان و به‌هیزی و هیزی قوولی ئه‌لگۆریتمه‌که‌ی پشتره‌ست ده‌کاته‌وه‌..

کلیله وشه: داره‌ که‌مترین په‌ستانی دوو نامانج (MST)، ئه‌لگۆریتمیکی تیکه‌ل، داره‌ که‌مترین په‌ستانی سنووردار، باشکردنی دیزاینی تۆر، ههمه‌چه‌شه‌نی پێشه‌وه‌ی پارێتو.

1. INTRODUCTION

The design of efficient networks is a fundamental task in operations research and computer science, with applications in communication systems, transportation, logistics, and energy distribution. The minimum spanning tree (MST) is one of the most studied models in this area because it provides a cost-efficient way of connecting nodes in a graph. However, real-world problems rarely involve a single criterion. Instead, decision-makers must simultaneously consider multiple objectives such as minimizing construction costs, reducing delays, and improving reliability. This requirement leads to the bi-objective or multi-objective MST problem, which has received increasing attention in recent years [1] [2]. Several methods have been proposed to extend classical MST models to multi-objective and constrained contexts. For example, exact approaches using integer programming or ϵ -constraint formulations have been applied to bi-objective MSTs, showing their effectiveness in generating Pareto-optimal solutions [2] [3]. Nevertheless, such approaches are limited in scalability and often cannot handle larger instances [4]. Other studies investigated uncertainty within the MST framework, highlighting that when edge costs or delays are imprecise, the optimization process becomes significantly more complex [5]. These findings underline the fact that although exact formulations are valuable, they remain unsuitable for real-world, large-scale constrained networks. Consequently, much research has shifted toward evolutionary and metaheuristic algorithms. Evolutionary algorithms have been widely used to approximate Pareto fronts and provide flexible trade-off solutions [6] [7]. Time complexity analyses of such algorithms confirm their strengths but also reveal limitations in convergence speed and feasibility maintenance [8]. In addition, new bio-inspired algorithms such as artificial rabbits optimization [9] and whale optimization [10] have shown their effectiveness across engineering problems, further motivating their use in spanning tree optimization. Despite these improvements, metaheuristics frequently suffer from premature convergence, reduced diversity, and difficulties in balancing exploration with exploitation.

To address these weaknesses, researchers have explored hybrid approaches that combine global exploration with problem-specific exploitation. Hybrid frameworks for spanning trees demonstrate significant improvements by incorporating heuristic seeding, genetic operators, or local search into evolutionary algorithms [7]. In addition, geometry-aware evolutionary methods have recently emerged as a powerful tool. Adaptive geometry-based algorithms explicitly model the shape of the Pareto front, enabling better distribution of solutions on irregular or non-convex trade-offs [11]. Moreover, new bio-inspired algorithms such as multi-objective crested porcupine optimization have achieved strong performance by integrating multiple operators for exploration and exploitation [12]. At the same time, recent advances in many-objective optimization with hybrid mechanisms confirm that integrating diverse forces can provide robustness and scalability [13]. Nevertheless, significant gaps remain. First, exact algorithms are constrained to small instances [4]. Second, existing evolutionary methods often lose Pareto diversity under strict feasibility rules [6], [5]. Third, although hybridization has improved performance, there is still a lack of integrated designs that combine exploration-biased bio-inspired methods with geometry-based survival strategies while also enforcing feasibility through repair operators.

In response to these challenges, this paper introduces a Hybrid MOCPO–AGE–MOEA, which integrates the exploration-rich operators of multi-objective crested porcupine optimization with the geometry-aware survival mechanisms of adaptive geometry-based evolutionary algorithms. The proposed method alternates between exploration and exploitation, applies operator cross-injection, and employs feasibility-first repair to guarantee valid spanning trees.

The contributions of this study are threefold:

- A formal problem formulation of the constrained bi-objective MST problem with hop and weight bounds.
- A novel hybrid algorithm combining bio-inspired exploration and geometry-aware survival, supported by repair mechanisms to maintain feasibility.
- A comprehensive experimental study on Euclidean graphs of various sizes, showing that the proposed method achieves superior Pareto diversity and hop minimization while maintaining competitive runtime.

The remainder of this paper is organized as follows: Section 2 reviews related works on MST optimization and hybrid evolutionary algorithms. Section 3 presents the mathematical problem formulation, including objectives, constraints, and evaluation rules. Section 4 explain background knowledge. Section 5 explains the proposed Hybrid MOCPO–AGE–MOEA framework, detailing the operator design, hybridization strategy, and selection process. Section 6 describes the experimental methodology, parameter settings, and performance metrics. Section 7 presents the experimental results and discussions. Finally, Section 8 concludes the paper and highlights directions for future research.

2. RELATED WORKS

Research on constrained and Mult objective spanning tree problems has progressed through both exact formulations and evolutionary approaches. To begin with, early contributions focused on exact mathematical models that guarantee optimal solutions under multiple objectives. For example, Carvalho and Coco [3] addressed the bi-objective constrained minimum spanning tree (MST) problem by developing efficient formulations that balance cost and hop limits. Similarly, Carvalho and Ribeiro [14] introduced an exact bounded-error calibration tree approach that improved modeling accuracy, but such exact methods often face scalability issues when problem size increases. In addition, theoretical analyses have provided further insights; Shi, Neumann, and Wang [15] analyzed the time complexity of evolutionary algorithms for hop-constrained MST problems, showing how operator design influences convergence efficiency. Complementary to this, Carvalho [16] highlighted the importance of statistical evaluation methods when dealing with infeasible solutions in algorithmic experimentation, thereby improving the robustness of comparative studies. Moreover, Majumder et al. [5] examined Mult objective MSTs under uncertain conditions, extending the problem's applicability to real-world uncertain paradigms. In terms of specific hop-constrained formulations, Akgün and Tansel [17] proposed Miller–Tucker–Zemlin-based constraints to model hop limits more effectively, while de Sousa et al. [18] developed an exact bi-objective diameter-cost spanning tree formulation. Transitioning from theory to application, Wang et al. [19] investigated optimal tree topology in submarine cable networks under latency constraints, and Yamaoka et al. [20] introduced

MST-based methods for robust time-delay estimation in noisy communication systems. Additionally, Carvalho [21] explored complexity and relaxation techniques for hop-constrained MST problems, offering a deeper understanding of both theoretical and computational trade-offs. More recently, evolutionary computation has advanced with the introduction of dual-population strategies, as demonstrated by Geng et al. [22], which enhance performance in constrained many-objective problems. In parallel, Panichella [23] proposed improved Pareto-front modeling algorithms to strengthen diversity and convergence in large-scale scenarios. Qiao et al. [24] contributed scalable benchmark suites and algorithms for high-dimensional constrained optimization, which are crucial for testing new designs across diverse problem types. Finally, Zhang and Jin [25] emphasized the necessity of rigorous statistical evaluation of Mult objective evolutionary algorithms, confirming that reliable assessment frameworks are essential for advancing constrained optimization research. Altogether, these studies show a consistent trend: exact formulations provide valuable theoretical benchmarks, but scalable hybrid and evolutionary methods, supported by dual-population strategies, geometry-aware modeling, and rigorous statistical testing, are now essential to achieve both feasibility and efficiency in constrained bi-objective MST optimization.

3. STUDY FRAME WORK

This section defines the constrained bi-objective Minimum Spanning Tree (MST) problem and the mathematical formulations that guide algorithm design, ensure fair evaluation, and support reproducibility [4] [26] [27].

$$\min F(T) = (f_1(T), f_2(T)) \quad (1)$$

Where: T = spanning tree, $f_1(T)$ = total weight, $f_2(T)$ = maximum hop count.

$$f_1(T) = \sum_{(u,v) \in T} w(u,v) \quad (2)$$

Where: $w(u,v)$ = Euclidean weight of edge (u,v) , T = set of tree edges.

$$f_2(T) = \max_{v \in V} \text{dist_BFS}(r, v; T) \quad (3)$$

Where: r = root node, v = vertex, $\text{dist_BFS}(r,v;T)$ = BFS distance from r to v .

$$|T| = |V| - 1 \quad (4)$$

Where: $|V|$ = number of vertices, $|T|$ = number of edges in tree.

$$f_1(T) \leq W_{\max}, f_2(T) \leq H_{\max} \quad (5-6)$$

Where: W_{\max} = maximum allowed weight, H_{\max} = maximum allowed hop depth.

$$\text{fit}(T) = (f_1(T), f_2(T)) \quad (7)$$

Where: $\text{fit}(T)$ = fitness vector, f_1, f_2 = weight and hop objectives.

$$\text{fit}(T) = (+\infty, +\infty) \quad (8)$$

Where: $+\infty$ = sentinel for constraint violation, ensuring dominance loss.

$$A < B \Leftrightarrow \forall i, fi(A) \leq fi(B) \wedge \exists j, fj(A) < fj(B) \quad (9)$$

Where: A, B = two solutions, fi = objective function, i, j = indices.

$$z * i = \min_{x \in P} fi(x) \quad (10)$$

Where: $z * i$ = best value of objective i , P = population.

$$fi'(x) = (fi(x) - z * i) / (\max_j fi(j) - z * i) \quad (11)$$

Where: $fi(x)$ = objective value of solution x , $\max_j fi(j)$ = maximum value in population.

$$dx = \left(\sum_{i=1}^m (fi'(x))^p \right)^{1/p} \quad (12)$$

Where: $d(x)$ = distance, m = number of objectives, p = norm parameter (commonly 2, Euclidean distance).

$$S(x) = \lambda \cdot (1/d(x)) + (1 - \lambda) \cdot \text{spread}(x) \quad (13)$$

Where: $S(x)$ = survival score, λ = balance factor, $d(x)$ = distance to ideal, $\text{spread}(x)$ = diversity measure.

Together, these formulations define the constrained MST problem and the mechanisms underlying the hybrid MOCPO–AGE-MOEA. They provide a rigorous mathematical foundation for solution construction, evaluation, and selection, ensuring that the experimental framework is both transparent and reproducible.

5. THE PROPOSED HYBRID OPTIMIZATION ALGORITHM

As depicting from below flowchart, the proposed hybridization between **MOCPO** and **AGE-MOEA** operates through a multi-level integration that balances exploration and exploitation in equal proportion (50%–50%) at every stage of the evolutionary cycle. we explored many ratios through a set of preliminary experiments to balance exploration and exploitation. The 50–50 configuration was ultimately selected because it consistently produced more stable results across different graph sizes, preserving both Pareto diversity (driven by MOCPO) and convergence quality (guided by AGE-MOEA). First, the process begins with graph construction, root node selection, and a diversified population initialization strategy that combines greedy Kruskal-based solutions, random spanning trees, and BFS-biased trees. This initialization ensures that both objectives, cost minimization and hop efficiency, are represented from the start while also guaranteeing structural diversity. Subsequently, the algorithm enters its iterative phase, where the scheduling mechanism alternates between the two engines: during even iterations, the MOCPO module is executed, whereas during odd iterations, the AGE-MOEA module is activated. This alternation establishes a regular cadence, allowing each paradigm to influence the search process in successive generations.

In the **MOCPO step**, the search is divided evenly between exploration and exploitation. On the one hand, exploration operators (Sight and Sound) account for 50% of offspring generation and encourage structural variety by recombining edge sets and completing them with randomized spanning-tree finishers. On the other hand, exploitation covers the remaining 50% and is itself hybridized: half of this share uses native MOCPO intensification operators (Odor and Physical), which gradually drive solutions toward the current best by scaling edge-adoption probabilities through fitness ratios and time-dependent cooling; meanwhile, the other half applies a GA-based fallback mechanism, performing crossover with the best feasible solution followed by light mutation and Kruskal completion. In contrast, the **AGE-MOEA step** also maintains a 50%–50% balance: half of the offspring are generated by native GA operations (tournament selection, crossover, and mutation), while the other half leverage a MOCPO intensifier, where one child is refined through Odor/Physical exploitation and the other is produced via GA crossover with best feasible. Thus, in every iteration, whether MOCPO or AGE-MOEA is the controlling engine, both exploration and exploitation are proportionally balanced, and moreover, each paradigm borrows operators from the other, ensuring cross-injection of strategies.

After offspring are produced, the hybrid consistently applies a **feasibility-first evaluation**, discarding infeasible solutions by assigning them infinite objective values. This is followed by **non-dominated sorting**, which organizes the population into Pareto fronts, and by **AGE-style survival selection**, which replaces NSGA-II's crowding distance with geometry-aware scoring. In this step, the algorithm normalizes objectives, preserves extremes, and evaluates survival scores using L^p norms ($p \approx 2$), thereby maintaining a well-distributed Pareto set across convex and concave fronts. The archive is updated accordingly, and the cycle continues until the termination condition is met. Through this multi-level hybridization, at the scheduler level (alternating engines), at the operator level (cross-injected exploration and exploitation), and at the selection level (geometry-aware survival), the algorithm combines the exploratory breadth and feedback-driven intensification of MOCPO with the geometry-adaptive preservation and refinement of AGE-MOEA. Consequently, the hybrid maintains diversity, converges efficiently, and respects feasibility constraints, ultimately yielding robust and well-spread Pareto-optimal solutions.

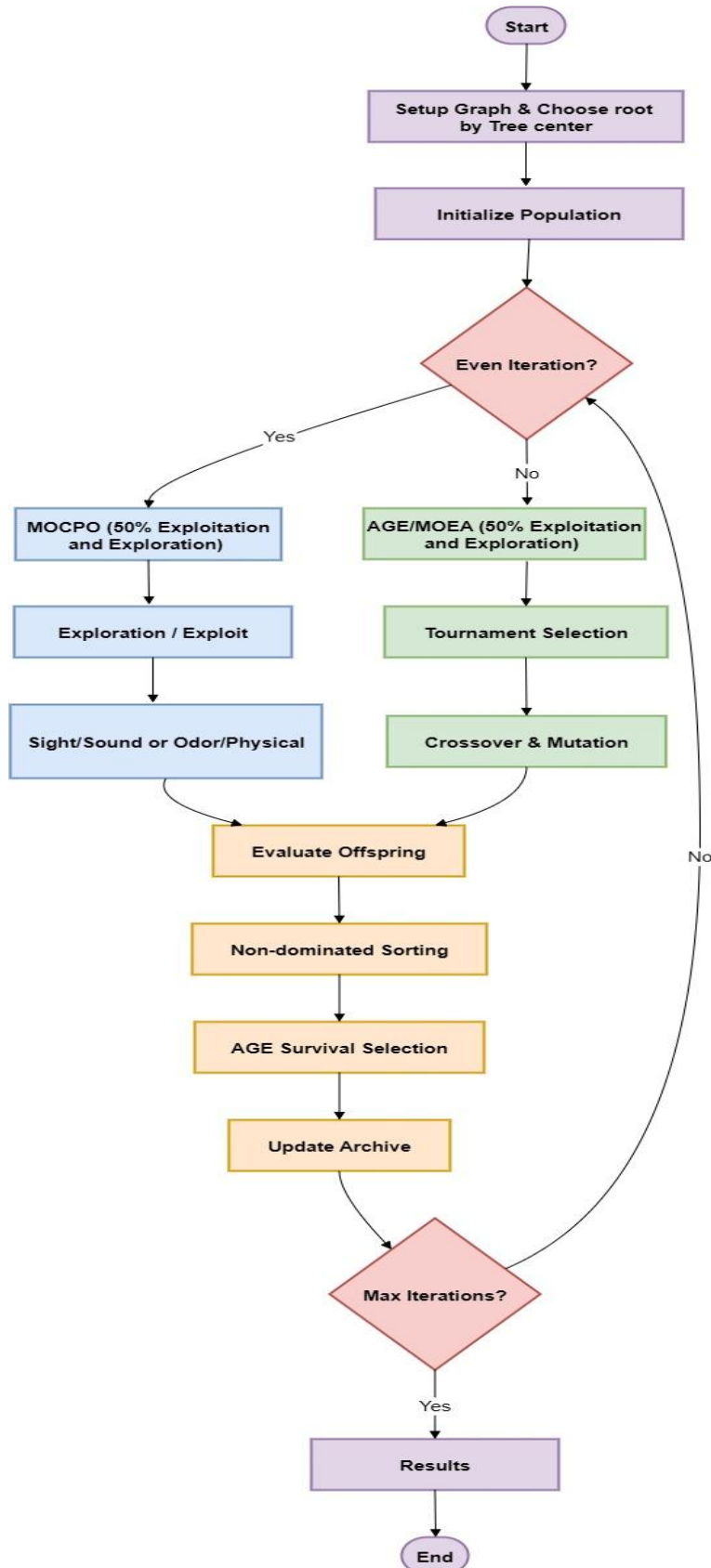


Figure 1. Flowchart of the hybrid optimization algorithm.

Figure 2 shows the step-by-step instructions of pseudo code for our hybrid algorithm that combines MOCPO and AGE-MOEA to solve the two-objective minimum spanning tree problem:

First, the algorithm builds a complete Euclidean graph from the given coordinates and, then, selects a root according to the specified policy (Tree-Center or Tree-Edge). Next, it initializes a diverse population with three seeds, greedy/Kruskal (cost-lean), random/Kruskal-biased (diversity), and BFS-heuristic (hop-lean), and immediately evaluates each tree with respect to the root to obtain total weight and maximum hops; afterward, it computes the current ideal point from the best feasible values. Subsequently, the main loop iterates up to maximum iteration: if the iteration is even, the **MOCPO step** generates one child per parent; specifically, with probability ≈ 0.45 it performs exploration via **Sight** or **Sound** followed by randomized completion, otherwise it performs exploitation where, in turn, half the time it applies native **Odor/Physical** moves (fitness/time-scaled) and half the time it uses a **GA fallback** (crossover with the best feasible plus light mutation and Kruskal completion). Meanwhile, every produced child is evaluated immediately and inserted into the offspring set.

Conversely, if the iteration is odd, the **AGE-MOEA step** first selects parents by tournament ($k=3$) and then, for each pair, produces two children; specifically, with probability 0.5 it follows the native GA path (crossover or copy, then mutation), otherwise it uses a **MOCPO-intensified** path where the first child is refined by Odor/Physical and the second is a light-GA child crossed with the best feasible. Then, parents and offspring are merged, duplicates are removed, and, crucially, feasibility-first **non-dominated sorting** forms Pareto fronts; if the last front overflows, the fill procedure applies **AGE-MOEA survival on the last front** (preserve extremes, normalize, and rank by L^p proximity/spread) to choose survivors. Finally, the ideal point is updated and iteration statistics are logged; after all iterations, the algorithm extracts the final Pareto front from the archive and, ultimately, returns this set together with summary metrics. The following notations are used in Figure 2 (Pseudo-code of the hybrid algorithm): G is the input graph and r is the chosen root, with s denoting a candidate spanning tree. The evolving sets are POP (population), OFF (offspring), UNI (their union), FRONTS (non-dominated layers), and PF_final (final Pareto front). Objectives are f_1 (weight) and f_2 (hops), constrained by W_{\max} and H_{\max} , while z^* marks the ideal point; the search runs for MAX_ITERS iterations with POP_SIZE solutions, and each solution is evaluated by evaluate (s , r , W_{\max} , H_{\max}).

INPUT: coords or n, POP_SIZE, MAX_ITERS, W_max, H_max, root_policy $\in \{TC, TE\}$

--- Graph construction ---

$G \leftarrow \text{build_graph}(\text{coords or } n)$

$r \leftarrow \text{select_root}(G, \text{policy} = \text{root_policy})$

--- Initialization ---

$\text{POP} \leftarrow \text{init_population}(G, r, \text{POP_SIZE})$ # greedy, random, BFS seeds

for s in POP:

$\text{evaluate}(s, r, W_max, H_max)$

ideal $\leftarrow \text{update_ideal}(\text{POP})$ # z^* = best values so far

--- Main loop ---

for it = 0 .. MAX_ITERS-1:

 OFF $\leftarrow \emptyset$

 if it is even: # ===== MOCPO step =====

 for parent in POP:

 if $\text{rand}() < 0.45$:

$\text{child} \leftarrow \text{sight_or_sound}(\text{parent}, \text{POP})$

$\text{child} \leftarrow \text{complete_with_kruskal_if_needed}(\text{child}, G)$

 else:

 if $\text{rand}() < 0.5$:

$\text{child} \leftarrow \text{odor_or_physical}(\text{parent}, \text{POP}, \text{ideal}, \text{it}, \text{MAX_ITERS})$

 else:

$\text{child} \leftarrow \text{ga_fallback}(\text{parent}, \text{best_feasible}(\text{POP}))$

$\text{evaluate}(\text{child}, r, W_max, H_max)$

 OFF $\leftarrow \text{OFF} \cup \{\text{child}\}$

 else: # ===== AGE-MOEA step =====

 mates $\leftarrow \text{tournament}(\text{POP}, k = 3)$

 for (p1, p2) in pairs(mates):

 if $\text{rand}() < 0.5$:

$(c1, c2) \leftarrow \text{crossover_or_copy}(p1, p2)$

$c1 \leftarrow \text{mutate}(c1); c2 \leftarrow \text{mutate}(c2)$

 else:

$c1 \leftarrow \text{odor_or_physical}(p1, \text{POP}, \text{ideal}, \text{it}, \text{MAX_ITERS})$

$c2 \leftarrow \text{light_ga}(p2, \text{best_feasible}(\text{POP}))$

$\text{evaluate}(c1, r, W_max, H_max)$

$\text{evaluate}(c2, r, W_max, H_max)$

 OFF $\leftarrow \text{OFF} \cup \{c1, c2\}$

```
# --- Environmental selection ---
UNI ← dedup(POP ∪ OFF)
fronts ← nondominated_sort(UNI, feasibility_first = True)
POP ← fill_fronts(fronts, POP_SIZE,
    last_front_tiebreak = age_moea_survival_on_last_front(ideal))

ideal ← update_ideal(POP)
log_iter(POP)                                # PF size, avg f1, avg f2

# --- Output ---
PF_final ← pareto_front_from_history()
return PF_final, metrics(PF_final)
```

Figure 2. Pseudo-code of the hybrid algorithm.

6. EXPERIMENTAL DESIGN AND SETUP

This section presents the experimental setup, including benchmarks, metrics, parameters, and baselines, to ensure fair and reproducible evaluation of the proposed algorithm.

This study evaluated the performance of six state-of-the-art multi-objective algorithms for the bi-objective constrained Minimum Spanning Tree problem to ensure comprehensive algorithmic comparison. The proposed Hybrid MOCPO-AGE-MOEA algorithm was tested against five established methods: MOCPO (2025), AGE-MOEA (2019), MOANA (2024), CoMMEA (2023), and MOMPA (2021). All algorithms were executed under identical experimental conditions to eliminate bias and ensure that performance differences were solely due to algorithmic design rather than environmental factors. The experimental design employed a systematic factorial approach with two root selection strategies (TC tree-center at coordinate 20,20 and TE tree-edge at coordinate 0,0) to evaluate robustness across different network topologies. Graph complexity was systematically varied across 50 sizes from 11 to 60 nodes with linear increments, and 50 independent runs were performed for each algorithm-configuration-size combination, resulting in 30,000 total experiments (6 algorithms × 2 configurations × 50 graph sizes × 50 runs). All algorithms used uniform parameters including population size of 50, maximum iterations of 50, weight constraint of 400.0, and hop constraint of 40, ensuring fair comparison across identical search spaces and computational budgets. A comprehensive configuration table with 16 standardized parameters organized into seven categories including population control, constraints, hybrid blending ratios, and algorithm-specific settings for both MOCPO and AGE-MOEA components. These uniform parameter values ensure fair algorithmic comparison by eliminating configuration bias, guaranteeing that any observed performance differences are solely attributable to the intrinsic capabilities of each algorithm rather than parameter variations.

Table 1 Algorithm Parameters

Group	Parameter	Meaning	Typical
General	POP_SIZE	population size	50
	MAX_ITERS	iterations per run	50
	W_max,H_max	hard constraints	400, 40
	root_policy	TC or TE	TC (default)
Completion	complete_mode	Kruskal / randomized (both used)	mixed
GA	crossover_rate	prob. of crossover	0.9
	mutation_rate	per-child mutation prob.	0.1–0.2
	tournament_k	tournament size	2–3
MOCPO	explore_prob	share for Sight/Sound	~0.45
	exploit_split	native vs GA fallback in exploitation	50/50
Hybrid	even_iter_engine	MOCPO on even iters	fixed even_iter
	odd_iter_engine	AGE/EA on odd iters	fixed odd_iter
AGE	p_norm	Minkowski p for survival	2

Table 1 summarizes the key parameters used in the hybrid optimization framework, covering general settings, GA operators, MOCPO behavior, hybrid scheduling, and AGE survival. The general parameters define population size, iteration count, and feasibility constraints, while the GA parameters control crossover, mutation, and tournament selection. Meanwhile, MOCPO and AGE settings regulate the balance between exploration and exploitation, with the hybrid alternating engines by iteration to ensure both diversity and convergence.

Table 1 Evaluation Criteria

Metric	Direction	Description
Average Pareto Front Size	↑	Number of non-dominated solutions (diversity)
Average Execution Time	↓	Computational efficiency (seconds)
Average Weight	↓	Cost-effectiveness of solutions
Average Hops	↓	Communication efficiency (latency)

Table 2 includes evaluation criteria assess different aspects of algorithm performance with clear optimization directions. Higher Pareto Front Size indicates better solution diversity and trade-off exploration, while lower values for the other three metrics represent superior performance in speed, cost, and communication efficiency respectively. The arrows (↑↓) indicate whether higher or lower values are preferred for each metric.

This study used a three-step statistical method to make fair and reliable comparisons between the algorithms. First, the Friedman test was applied to detect overall performance differences across all metrics, since it is a non-parametric method suitable for repeated measures across multiple algorithms. Second, the Bonferroni correction was used to adjust pairwise comparisons, thereby reducing the risk of Type I errors when interpreting significance across multiple algorithm pairs. Third, the Cohen's d (α) alpha was calculated to quantify the practical importance of performance differences beyond statistical significance. For interpretation, Cohen's $d < 0.2$ was considered

negligible, 0.2–0.5 small, 0.5–0.8 medium, and ≥ 0.8 large. Collectively, this framework ensures that the reported results capture both statistical reliability and practical impact.

7. Performance Evaluation and Critical Analysis

This section covers the experimental results and statistical analyses, highlighting the comparative performance of the Hybrid MOCPO-AGE-MOEA against five state-of-the-art algorithms in terms of Pareto diversity, hop minimization, runtime efficiency, and solution weight.

Figure 3 presents the Friedman test results expressed as $-\log_{10}(\text{p-values})$ across four evaluation metrics: Pareto front size, execution time, average weight, and average hops. The dashed line indicates the $p = 0.05$ significance threshold, and all bars rise dramatically above this line, with values ranging from 10^{-40} to 10^{-48} . This demonstrates that the observed performance differences among algorithms are extremely significant and not due to random variation. Pareto front size shows the strongest evidence with $p = 1.33 \times 10^{-48}$, execution time is nearly as strong with $p = 5.16 \times 10^{-47}$, and both average weight ($p = 7.81 \times 10^{-44}$) and average hops ($p = 8.27 \times 10^{-40}$) also indicate highly significant differences. Overall, the figure confirms that all four metrics provide systematic and meaningful evidence of algorithmic variation.

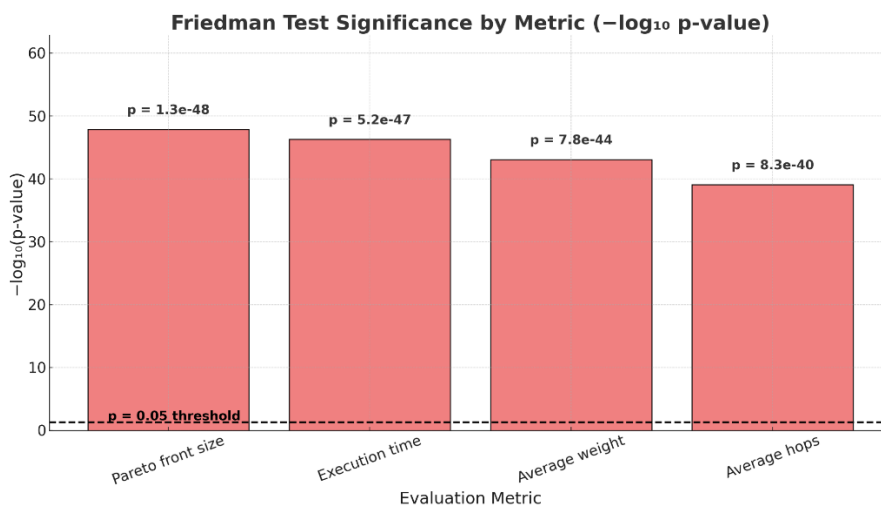


Figure 3. Friedman Test Significance by Metric ($-\log_{10} \text{p-value}$)

Figure 4 shows the χ^2 statistics for the same four evaluation metrics, with larger values reflecting stronger evidence of differences across algorithms. The results range from $\chi^2 = 193.15$ to 234.23 , confirming that each metric exhibits substantial variation. Annotations highlight the relative performance, where the Hybrid MOCPO-AGE-MOEA achieved the best Pareto front size and lowest hop count, while MOCPO dominated execution time and average weight. In contrast, MOMPA was weakest in diversity, CoMMEA performed poorly in runtime, and the Hybrid showed a trade-off with heavier weights despite excelling in hops. Together, the results illustrate a clear division of strengths: MOCPO is efficient and lightweight, while the Hybrid is superior in diversity and latency reduction, with MOMPA and CoMMEA emerging as the weakest performers.

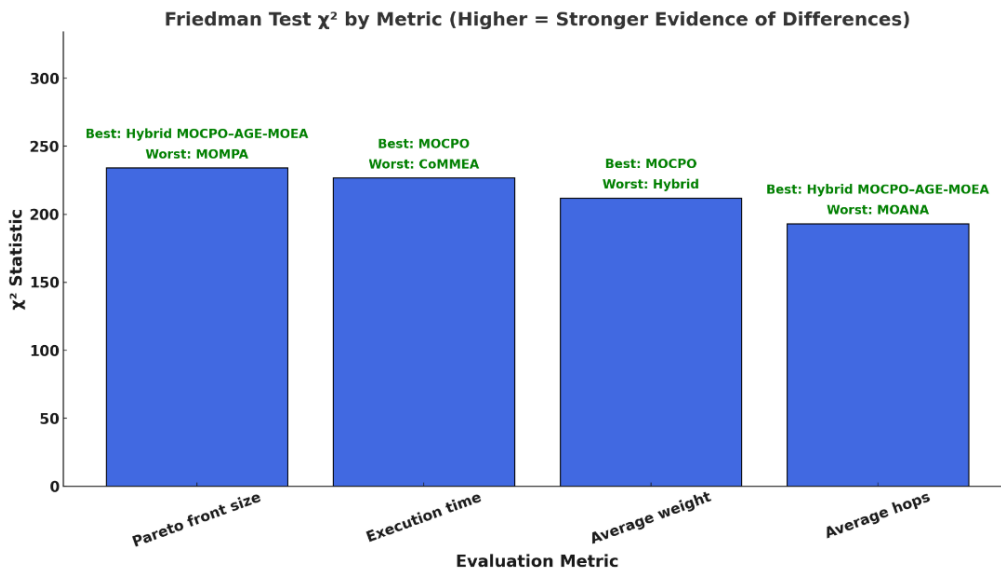


Figure 4. Friedman Test χ^2 by Metric (Higher = Stronger Evidence of Differences)

The Friedman ranking analysis in Figure 5 provides a comprehensive comparison of algorithmic performance across multiple evaluation metrics. The Hybrid MOCPO-AGE-MOE achieved the top rank in Pareto front size (1.00) and average hops (1.00), demonstrating its ability to generate diverse sets of trade-off solutions while simultaneously minimizing communication delay, two properties that are particularly important for constrained MST applications in time-sensitive domains. In addition, the Hybrid maintained a competitive runtime (1.66), nearly identical to MOCPO's best score (1.64), indicating that its improvements in diversity and latency reduction do not incur significant computational overhead. The only dimension where the Hybrid ranked lower was average weight (5.98), reflecting a deliberate trade-off between cost efficiency and enhanced solution diversity with reduced latency. By contrast, MOCPO performed best in weight and runtime but exhibited weaker results in diversity and hops, while other algorithms such as CoMMEA and MOMPA consistently ranked lower across most metrics. Overall, the Friedman analysis confirms that the Hybrid provides the most balanced and practically scalable performance, offering a strong combination of efficiency, diversity, and latency minimization that outperforms competing approaches.

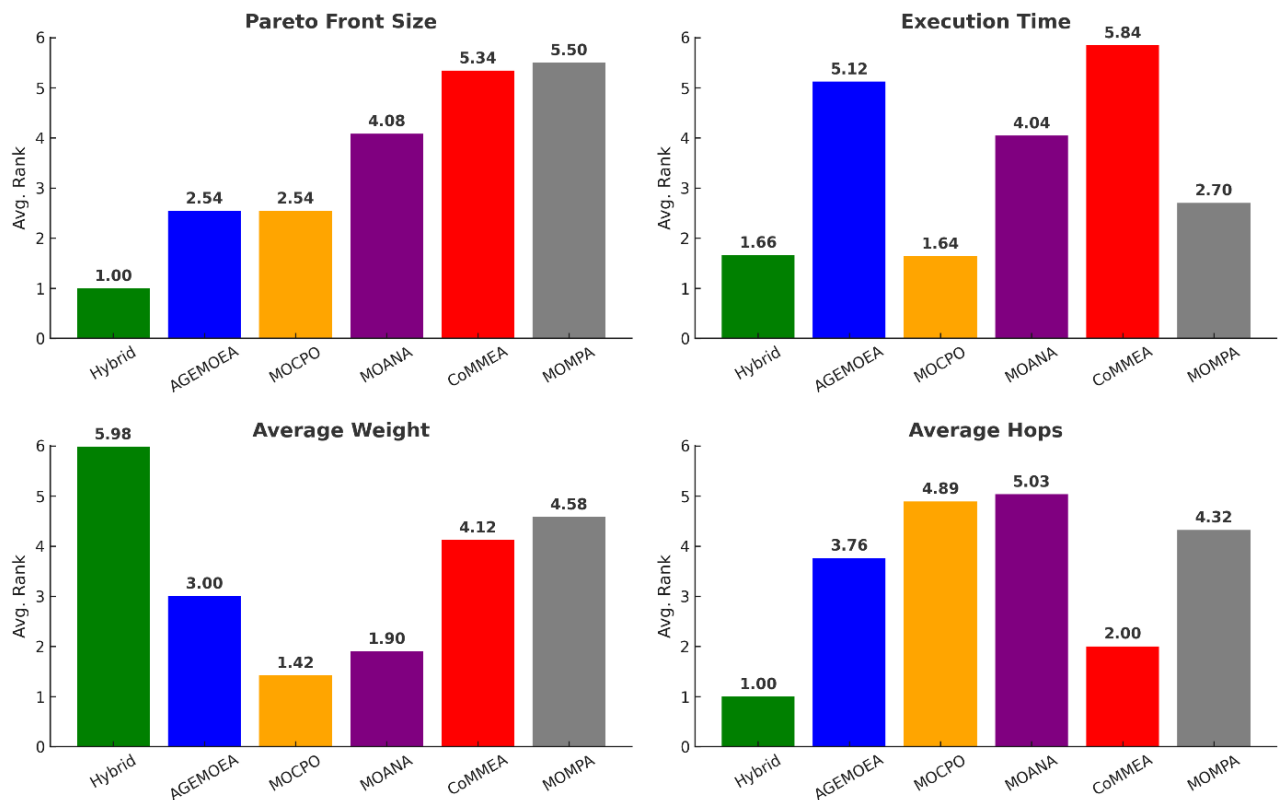


Figure 5. Friedman Ranking Analysis of Algorithms Across Performance Metrics

Table 3 shows that the Hybrid MOCPO-AGE-MOEA achieved the best overall rank (2.41), confirming its superiority across most metrics. MOCPO followed closely in second place (2.62), highlighting its strength in runtime and weight despite weaker diversity. The remaining algorithms, AGEMOEA, MOANA, MOMPA, and CoMMEA, ranked lower, indicating less balanced performance overall.

Bonferroni analysis confirmed strong pairwise differences. Out of 15 comparisons, 14 were significant for Pareto size, all 15 for runtime, nine for weight, and 13 for hops. Non-significant cases occurred mainly between AGEMOEA and others (MOCPO, MOANA, CoMMEA).

Table 3. Overall Rankings

Rank	Algorithm	Avg. Rank
1	Hybrid MOCPO–AGE-MOEA	2.41
2	MOCPO	2.62
3	AGEMOEA	3.61
4	MOANA	3.76
5	MOMPA	4.28
6	CoMMEA	4.33

Descriptive results (Table 4) confirmed that the Hybrid produced the largest Pareto size (151.85), nearly three times competitors, but at the cost of higher weight (178.64). It also achieved the lowest

hops (11.33), while MOCPO remained fastest (1.72s). CoMMEA achieved competitive hops but suffered from extreme runtime (20.60s).

Table 4. Performance Results for 50-Node Graph Instances

Algorithm	Avg. Pareto ↑	Avg. Hops ↓	Avg. Time ↓	Avg. Weight ↓
Hybrid	151.85	11.33	1.77	178.64
MOCPO	50.00	14.75	1.72	158.29
AGEMOEA	50.00	14.29	7.56	160.13
MOANA	47.25	14.73	4.51	158.65
MOMPA	39.23	14.55	2.33	162.20
CoMMEA	43.06	13.02	20.60	161.81

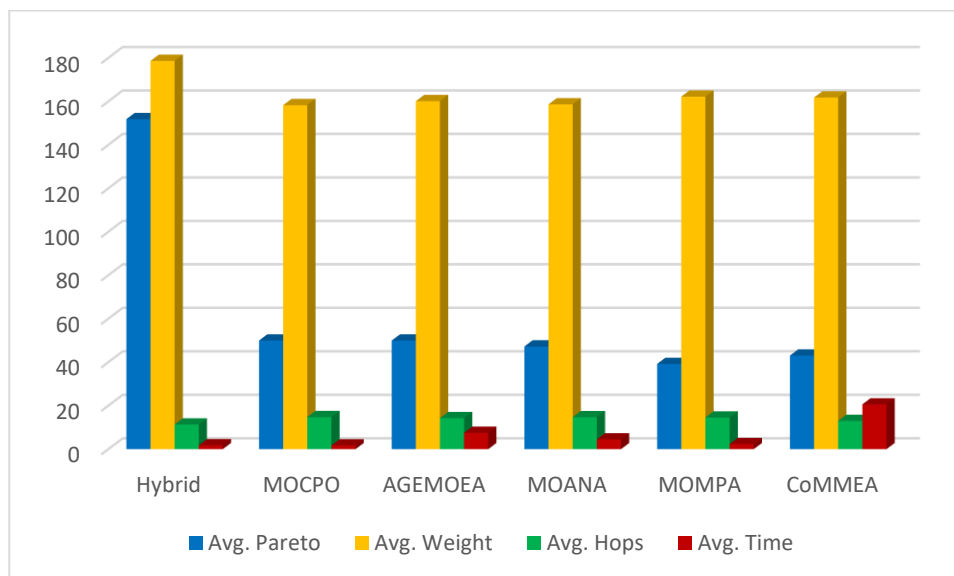


Figure 6. Descriptive Performance (Mean) for 50-Node Instances

Figure 6 and Table 4 clearly show that the Hybrid MOCPO–AGE-MOEA dominates in Pareto front size with an average of 151.85, which is nearly three times greater than all other algorithms. It also achieves the lowest hops (11.33), reflecting superior communication efficiency, though at the expense of higher weight (178.64) compared to the others. MOCPO provides the best performance in terms of execution time (1.72s) and weight (158.29), but its Pareto diversity is limited to 50. By contrast, CoMMEA demonstrates competitive hop reduction (13.02) but suffers from extremely high runtime (20.60s), highlighting its lack of practical efficiency.

Table 5 and Figure 7 present the performance results for 70-node graph instances and reveal consistent patterns with even clearer contrasts among the algorithms. The Hybrid algorithm obviously dominates in Pareto front size (145.96), producing nearly triple the diversity of competitors, while also achieving the lowest hop count (10.99), highlighting superior communication efficiency. This comes at the cost of slightly higher weight (170.65) and moderate runtime (2.07s). By contrast, MOCPO is the fastest (1.29s) but fails to provide complete results for weight and hops, indicating instability at this scale. AGEMOEA and MOANA deliver balanced outcomes with reasonable weights (161.54 and 151.91, respectively) but are constrained by high hop counts (≈ 14.3 – 14.5).

MOMPA performs competitively in runtime (1.76s) yet suffers from the highest hop values (15.63). Finally, CoMMEA demonstrates incomplete results and extreme runtime (15.95s), limiting its practical usability despite achieving a moderate Pareto size (43.52).

Table 5. Performance Results for 70-Node Graph Instances

Algorithm	Avg. Pareto ↑	Avg. Hops ↓	Avg. Time ↓	Avg. Weight ↓
Hybrid	145.956	10.999	2.065	170.646
MOCPO	49.937	-	1.292	-
AGEMOEA	47.142	14.530	6.375	161.539
MOANA	47.335	14.301	5.520	151.908
MOMPA	36.484	15.627	1.757	169.675
CoMMEA	43.517	-	15.947	-

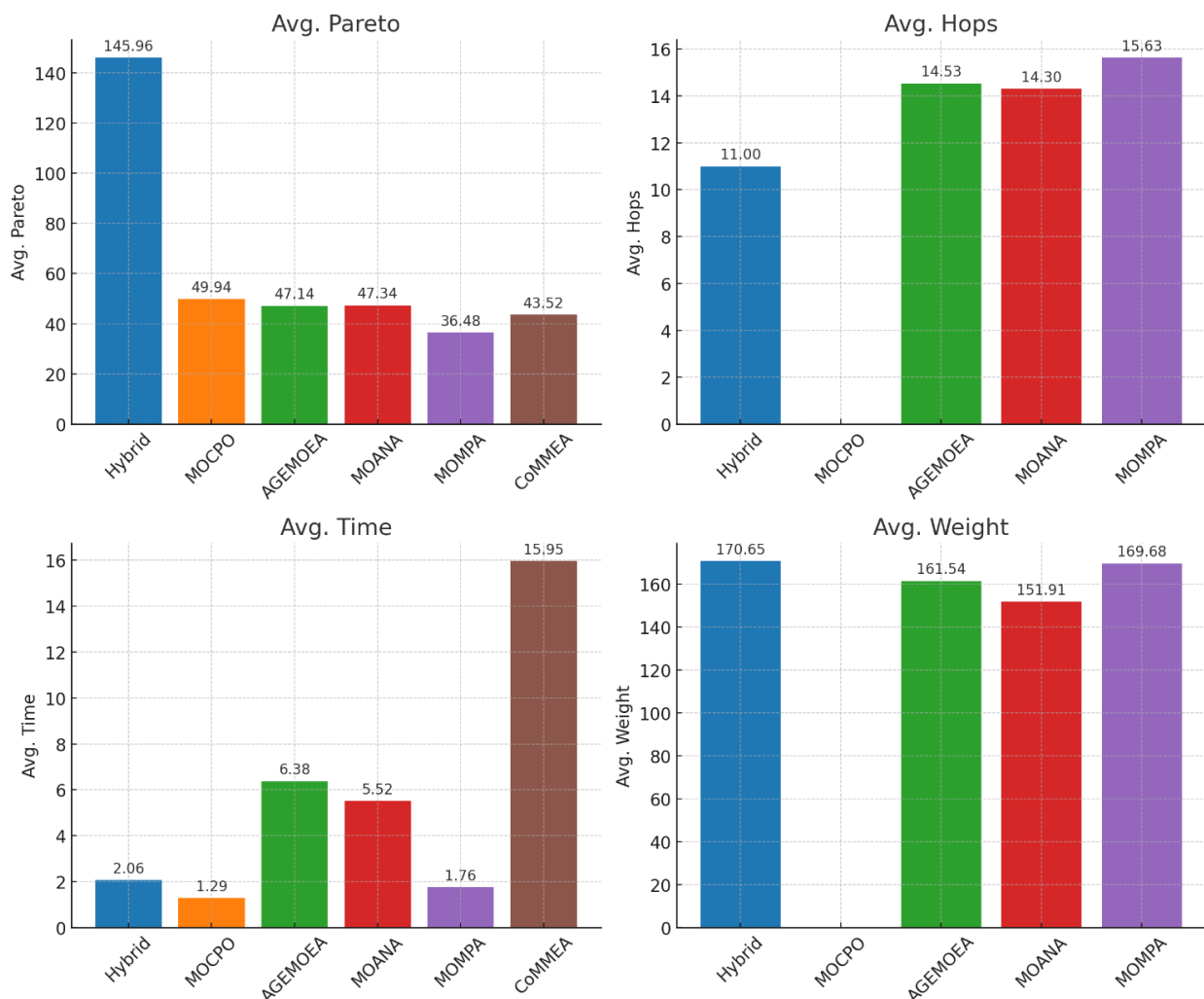


Figure 7. Performance (Mean) for 70-Node Instances Across all Algorithm

Cohen's d analysis (Table 5) showed that execution time (1.433s) and Pareto front size (1.386) had large practical importance, while weight (0.196) and hops (0.278) were statistically significant but practically small.

Table 5. Effect Sizes

Metric	Mean d	Interpretation
Execution time	1.433	Large
Pareto size	1.386	Large
Avg. hops	0.278	Small
Avg. weight	0.196	Small

7.2 Algorithmic Innovation

The key novelty of this work lies in the development of a hybrid evolutionary framework that strategically integrates the exploration capacity of MOCPO with the geometry-based survival strategy of AGE-MOEA. Unlike traditional evolutionary methods that often rely on crowding distance to maintain diversity, AGE-MOEA employs a geometry-aware approach: survival is guided by normalized L^p distances to the ideal point, preserving extreme solutions and ensuring a uniform spread across convex, concave, and irregular Pareto fronts. This mechanism directly addresses the loss of diversity and premature convergence that frequently occur in constrained MST optimization. Building on this principle, the proposed hybrid alternates MOCPO and AGE engines at successive iterations, employs operator-level cross-injection to strengthen adaptability, and applies feasibility-first repair to guarantee that hop and weight constraints are satisfied. The 50-50 ratio between MOCPO and AGE was determined through extensive preliminary experiments, where this configuration consistently achieved stable and well-balanced results across graph sizes. As a result, the hybrid achieves three major contributions: (i) uniting bio-inspired exploration with geometry-based survival in a single framework, (ii) introducing a feasibility-preserving repair mechanism that maintains constraint satisfaction without compromising solution quality, and (iii) delivering a balanced performance profile that combines Pareto diversity, hop minimization, and competitive runtime. Importantly, scalability experiments extended beyond the standard 50-node benchmarks to larger 70-node instances, where the Hybrid continued to dominate competitors by producing nearly triple Pareto front diversity and the lowest hop counts, thereby confirming its robustness, deep algorithmic strength, and suitability for real-world network design tasks.

A main result is the Hybrid algorithm's ability to reduce hop count, with an average of 11.33 compared to 13-15 achieved by competing methods. This improvement can be directly linked to the integration of MOCPO's exploitation-oriented operators (Odor and Physical) with AGE-MOEA's geometry-preserving survival, which together drive the search toward topologies that minimize communication delay. Although this came with a modest trade-off in weight (178.64 versus ~158 for MOCPO), the benefit is clear: lower hops correspond to reduced latency, which is often more critical in time-sensitive applications such as telecommunications, sensor networks, and distributed computing.

Runtime efficiency further confirms the practicality of the proposed approach. Despite its more complex operator schedule, the Hybrid maintained near-identical runtime to MOCPO (1.77s vs. 1.72s), showing that its gains in diversity and hops do not introduce significant computational overhead. This scalability is essential for real-world scenarios, where solutions must be generated

quickly even as network size grows. By contrast, methods such as CoMMEA achieved competitive hop values but required excessive runtime (20.60s), undermining their viability in practice.

Overall, the results highlight a clear division of strengths among the tested algorithms: MOCPO is strong in runtime and weight but weak in diversity, CoMMEA performs well in hops but fails in efficiency, while the Hybrid uniquely balances all four-evaluation metrics. This balance is what elevates the Hybrid as the most effective algorithm in the study, it not only generates feasible and efficient solutions but also expands the decision space available to practitioners. The combination of broad Pareto coverage, low latency, and competitive runtime establishes the Hybrid MOCPO-AGE-MOEA as a practical and scalable solution for constrained MST optimization, with significant potential for deployment in real network design tasks.

8. CONCLUSION

This study addressed the constrained bi-objective Minimum Spanning Tree problem by introducing a Hybrid MOCPO-AGE-MOEA that integrates exploration-driven operators with geometry-aware survival and feasibility-first repair. Extensive experiments on Euclidean graphs demonstrated that the Hybrid consistently outperforms state-of-the-art algorithms in two critical aspects: Pareto front diversity and hop minimization. On average, it generated nearly three times more non-dominated solutions than its closest competitor and reduced hops to 11.3, offering clear benefits for latency-sensitive networks. Although MOCPO remained strongest in execution time and weight, the Hybrid delivered a more balanced performance overall, maintaining competitive runtime while substantially expanding the decision space available to network designers.

These findings confirm that the Hybrid provides a superior trade-off between diversity, efficiency, and scalability, making it the most effective method among those tested. Beyond benchmark results, its ability to generate broad and high-quality Pareto sets ensures practical value for applications in communication, IoT, and logistics networks where both cost and responsiveness are critical. Future research will extend this work by testing the Hybrid on dynamic and real-world network instances, exploring parallelization strategies, and investigating adaptive mechanisms for operator scheduling to further improve efficiency.

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