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Comparison between Biexponential and Robust Biexponential Nonlinear Models, Using Simulation

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Abstract
The Science of statistics has become paramount importance in this age as a means and a tool for scientific method in research in all the various fields of science. Linear regression is a powerful method for analyzing data described by models which are linear in the parameters.

In this paper, we compared between nonlinear regression method and robust method. Nonlinear models tend to be used either when they are suggested by theoretical considerations or to build known nonlinear behavior into a model. Even when a linear approximation works well, a nonlinear model may still be used to retain a clear interpretation of the parameters. By using R language software we generate the data that we use in this paper for three sample sizes (25,50,100) and we took (200) reputations for each sample sizes. In the practical part of this thesis for Nonlinear Regression, we used Biexponential Nonlinear Regression Model and we estimate four parameters. We found the estimated parameters mean for our model, and we also found that the best performance for parameters by depending on the Akaike information criterion and Bayesian information criterion.We tested the parameters, so we found that they are significant also the ANOVA table where the F-test is significant.

The best model is Biexponential Robust Nonlinear Regression Model **by depending on the parameters mean** as an initial value from Biexponential Nonlinear Regression Model for sample size (100) because it has the minimum AIC and BIC.

Keywords*: Linear Regression, Nonlinear Regression, Robust Regression, Biexponential, Weight Functions, ANOVA, AIC, BIC.*

پوختە
زانستی ئامار گەشەیەکی زۆر و فراوانی کردووە لەم سەردەمە و گرنگیەکی زۆری هەیە و بووە بە ڕێگایێك یاخود ئامرازێك بۆ میتۆدی زانستی لە توێژینەوەکاندا لە هەموو بەشە جۆراوجۆرەکانی زانستدا، زانینی ئامار زانستێکە کە گەشەی سەندووە و گەشەی کردووە بۆ سەدەی ئێستاکە و بووە بە زانستێکی سەقامگیر، کە داڕشتە و یاسای خۆی هەیە.
لەم توێژینەوەیە دا بەردراوردمان کرد لەنێوان میتۆدی پاشۆچوونی نا-هێڵی و میتۆدی ڕۆبەست (پارێزبەند)، مۆدێلە نا-هێڵیەکان بەکار دەهێنرن کاتێک لە لایەن تێرامانی بیردۆزیەکانەوە پێشنیار دەکرێن یان لە کاتی بنیاتنانی ڕەفتارێکی زانراوی نا-هێڵی بۆ مۆدێلێك. تەنانەت ئەو کاتەی نزیککردەوەی هێڵیش بە باشی کار دەکات ، مۆدێلی نا-هێڵی لەوانەیە هێشتا بەکار بهێنرێت تاکو لێکدانەوەیێکی ڕوونی هۆکار و پێوەرەکان بهێلێتەوە. بە بەکارهێنانی زمانی سۆفتوێری R ئێمە زانیاریەکە دروست دەکەین بە شێوەیەک کە لەم توێژینەوەیە دا سێ قەبارەی جیاواز وەک نموونە بەکار دەهێنین کە بریتین لە (25، 50، 100)، و (200) جار پرۆسەی دووبارەکردنەوەمان بۆ هەر قەبارەیەک بەکارهێناوە. لە بەشی کرداری ئەم توێژینەوەیە دا بۆ لاری بوون (پاشۆچوونی) نا-هێڵی، مۆدێلی لاری بوون (پاشۆچوونی) نا-هێڵی بای -ئێکسپەونێنشەل (دوانەی توانی)مان

بەکارهێنا وە و خەملاندمان بۆ چوار دانە لە پێوەر و هۆکارەکان کرد و کۆی پێوانەی خەملاندنمان دۆزیەوە بۆ مۆدێلەکەمان وهەروەها باشترین جێبەجێکارمان بۆ هۆکار و پێوەرەکان دۆزیەوە بە پشت بەستن بە پێوانەی زانیاری Akaike وپێوانەی زانیاری Bayesian. وە تاقیکردنەوەمان بۆ پێوەرەکان کرد و بایەخداربوون وە بە هەمان شێوە خشتەی (ANOVA) مان دۆزیەوە کە تیایدا تاقیکردنەوەی (F-test) بایەخداربوو.
باشترین مۆدێل بریتییە لە مۆدێلی نا-هێڵی لاری بوون (پاشۆچوونی) ڕۆبەستی بای -ئێکسپەونێنشەل بە پشت بەستن بە تێکرای پێوەرەکان وەکو نرخێکی سەرەتای لە مۆدێلی لاری بوون (پاشۆچوونی) نا-هێڵی بای –ئێکسپەونێنشەل بۆ قەبارەی نموونەی (100) لەبەر ئەوەی کەمترین AIC و BIC.

الملخص:

أصبح علم الإحصاء له أهمية قصوى في هذا العصر كوسيلة وأداة ليس فقط للنهج العلمي في البحث لكن في جميع مجالات العلوم المختلفة. علم الاحصاء هو العلم الذي نما وتطور في القرن الحالي، وأصبح علما مستقرا له هيكله ونظامه الخاص.
في هذه البحث، قارننا بين طريقة الانحدار غير الخطية و الطريقة الحصينة و ذلك لايجاد وسيلة قوية نميل لاستخدامها إما عندما يقترح من قبل الاعتبارات النظرية أو لبناء السلوك المعروف غير الخطية من النموذج. حتى عندما نقرب نموذج الخطي، لا تزال نموذج غير الخطية يميل إلى الإبقاء و تعطينا تفسيراً واضحاً للمعلمات. بأستخدام البرمجيات اللغة R نحن نولد البيانات التي نستخدمها في هذه الرسالة لمدة ثلاثة أحجام من العينات (25، 50، 100)، وأخذنا (200) تكرارات لكل حجم من العينات. في الجزء العملي لهذه البحث للانحدار غير الخطية، نفسر نموذج الانحدار غير الخطية لـ(Biexponential) وتقدير المعلمات الأربع. لقد وجدنا تقدير المعلمات و نعني لنموذجنا، وبهذا وجدنا أيضا أفضل أداء للمعلمات بالاعتماد على معيار المعلومات Akaike ومعيار المعلومات النظرية الافتراضية (Bayesian). اختبرنا المعلمات حيث المعلمات كانت معنوية. و وجدنا جدول تحليل التباين (ANOVA) حيث كان اختبار (F-test) ايضا معنوية.
بعد مقارنة كل جزء عملي مع بعضها البعض، فإننا نستنتج أن نموذج الانحدار غير الخطية الحصينة لـ(Biexponential) بالاعتماد على الوسط المعلمات كقيمة البدائية في نموذج الانحدار غير الخطية لـ(Biexponential) هو أفضل نموذج لأنه لديه الحد الأدنى من AIC و BIC لحجم العينة (100).

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1. Introduction

The first and possibly the most important question about Nonlinear Model Estimate (**NLME**) is why would one want to use them. This question, of course, also applies to nonlinear regression models in general as does the answer: interpretability, parsimony, and validity beyond the observed range of the data.

An important task in statistics is to find the relationships among random variables, if any, that exist in a set of variables when at least one is random, being subject to random fluctuations and possibly measurement error. In regression problems typically one of the variables is often called the **response** or **dependent** variable which is of particular interest and is denoted by $\boldsymbol{y}$**.** The other variables $x_1, x_2, \dots, x_k$, usually called **explanatory, regressors,** or **independents** variables, are primarily used to predict or explain the behaviors of $\boldsymbol{y}$.

Classical statistics and econometrics are based on parametric models. Typically, assumptions are made on the structural and the stochastic parts of the model and optimal procedures are derived under these assumptions. Standard examples are least squares estimators in linear models and their extensions, maximum likelihood estimators, and the corresponding likelihood-based tests. Many classical statistic and econometric procedures are well-known for not being robust, because their results may depend crucially on the exact stochastic assumptions and on the properties of a few observations in the sample. These procedures are optimal when the assumed model is exactly satisfied, but they are biased and/or inefficient when small deviations from the model are present. The results obtained by classical procedures can therefore be misleading on real data applications.

2. The Nonlinear Regression Model

Nonlinear regression may be a confined and narrow topic within statistics. However, the use of nonlinear regression is seen in many applied sciences, ranging from biology to engineering to medicine and pharmacology.

A nonlinear regression model can be written:

$$Y_n = f(X_n, \theta) + \varepsilon_n \tag{1}$$

We used Biexponential Nonlinear Regression Model (SSbiexp) in this paper.

Where (SSbiexp) is selfStart model evaluates the biexponential model function and its gradient. It has an initial attribute that creates initial estimates of the parameters φ_1, φ_2, φ_3 and φ_4.

We can represent the Biexponential Model as follow:

$$y(x) = \varphi_1 \exp[-\exp(\varphi_2)\, x] + \varphi_3 \exp[-\exp(\varphi_4)\, x] \tag{2}$$

3. Robust Statistics (Robust Method)

The word "Robust" is loaded with many-sometimes inconsistent-connotations. We use it in a relatively narrow sense: for our purposes, robustness signifies insensitivity to small deviations from the assumptions. Robust regression is an alternative to least squares regression when data is contaminated with outliers or influential observations and it can also be used for the purpose of detecting influential observations. Primarily, we are concerned with distributional robustness: the shape of the true underlying distribution deviates slightly from the assumed model, usually the Gaussian law. This is both the most important case and the best understood one. Much less is known about what happens when the other standard assumptions of statistics are not quite satisfied and about the appropriate safeguards in these other cases.

4. Estimation Methods

The most important concerns of statistical science is data analysis in a study or scientific research and interpret the results obtained from them. Data interpretation is built on the basis of the rules and special methods and in some cases the researcher may face the problem in the data such as the move away sample data on the supposed distribution and due to the presence of outliers (anomalous) values or different distribution of the community under study for the supposed distribution, which this is due to deviations from assumptions made traditional methods like (OLS) to estimate the linear model parameters. Many researchers have worked to find more efficient methods that are not affected by deviations from specific assumptions. These methods called Robust methods were less affected by the status of data breaches as a condition of the user analysis. The robust estimators are close to the efficiency of the capabilities of ordinary least squares in case of verifying assumptions and better in the case of deviation from these assumptions, and it is suitable for a wide class of distributions in the estimation of linear model parameters. We can represent the Huber and Bisquare (Biweight) functions that used in this paper as follow:

Huber:

$$W(x,c) = \begin{cases} 1 & if\ |x| < c \\ \frac{c}{|x|} & otherwise \end{cases} \tag{3}$$

Bisquare:

$$W(x,c) = \begin{cases} (1-(\frac{x}{c})^2)^2 & if\ |x| < c \\ 0 & other\ wise \end{cases} \tag{4}$$

5. Model Selection Criteria

The two most popular covariance model selection criteria are AIC and BIC, short for an information criterion or Akaike information criterion and Bayes information criterion, respectively. AIC for a given model m is defined as:

$$AIC = -2L + 2m \tag{5}$$

$$BIC = -2L + 2\log(n) * m \tag{6}$$

Where:
L: log likelihood
m: the number of parameter in the model,
n: sample size

The model with the smallest value of AIC or BIC is selected as best. As can be inferred from the differing penalty functions, AIC tends to select models with more covariance parameters and BIC selects models with somewhat fewer covariance parameters.

6. Description and Generate of Data

By using (R) language (R version 3.2.4). The data come from a laboratory study on the pharmacokinetics of the drug indomethacin and we depend on Two observations $(\boldsymbol{x_1}, \boldsymbol{x_2})$ as an initial value to generate the data where x_1= (0.25,0.50,0.75,1,1.25) and x_2 = (2,3,4,5,6,8) since $\boldsymbol{x_1}$and $\boldsymbol{x_2}$ are time of Indomethacin administration we can see that plasma concentration of Indomethacin decrease by increasing Time of administration. We used runif ( ) to Generate random numbers from the uniform distribution where runif ( ) generates random

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deviates and we use three sample sizes; as follows: $(n = 25)$, $(n = 50)$ and $(n = 100)$ for each sample size we took (200) Reputations so we can use our data in a Nonlinear Regression and apply our data to Biexponential Nonlinear Regression Model, Biexponential Robust Nonlinear Regression Model and Robust M-Estimates (Huber and Bisquare (Biweight)).

Table (1): The value of Mean Parameters Estimation for Biexponential Nonlinear regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.9218028	1.2938830	0.9358788	-1.8676772
	1.9261400	1.3248406	0.9282851	-1.8947399
	1.9240320	1.3241795	0.9287286	-1.8960803

Table (2): The value of Best parameters for Biexponential Nonlinear Regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.7868610	1.273806	0.9499278	-1.786601
	1.864754	1.290247	0.9468054	-1.852526
	1.771918	1.279541	0.9661654	-1.851838

Table (3): The values of Akaike Information Criterion and Bayesian Information Criterion in Biexponential Nonlinear Regression Model.

Sample size	AIC	BIC
	-75.7905398	-57.7066843
	-128.5648466	-107.0152552
	-227.396284	-202.380957

Table (4): The Parameters Test for Biexponential Nonlinear Regression Model.

Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
25	φ_1	1.7868610	0.28051	6.370	8.07e-10 **
	φ_2	1.273806	0.19866	6.412	6.35e-10 **
	φ_3	0.9499278	0.06049	15.703	< 2e-16 **
	φ_4	-1.786601	0.10923	-16.356	< 2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
50	φ_1	1.864754	0.21034	8.865	<2e-16 **
	φ_2	1.290247	0.13872	9.301	<2e-16 **
	φ_3	0.9468054	0.04200	22.543	<2e-16 **
	φ_4	-1.852526	0.08008	-23.134	<2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
100	φ_1	1.771918	0.14787	11.98	<2e-16 **
	φ_2	1.279541	0.10393	12.31	<2e-16 **
	φ_3	0.9661654	0.03044	31.75	<2e-16 **
	φ_4	-1.851838	0.05665	-32.69	<2e-16 **

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Table (5): The ANOVA Table for Biexponential Nonlinear Regression Model.

Sample Size	S.O.V	D.f	S.S	M.S	F-test
25	Regression	4	41.59898	10.399745	239.1214
	Residual Error	271	11.78619	0.043491476	
	Uncorrected Total	275	53.38517		
Sample Size	S.O.V	D.f	S.S	M.S	F-test
50	Regression	4	81.97559	20.4938975	447.0368
	Residual Error	546	25.03075	0.045843864	
	Uncorrected Total	550	107.00634		
Sample Size	S.O.V	D.f	S.S	M.S	F-test
100	Regression	4	160.9054	40.22635	849.4348
	Residual Error	1096	51.90284	0.047356605	
	Uncorrected Total	1100	212.80824		

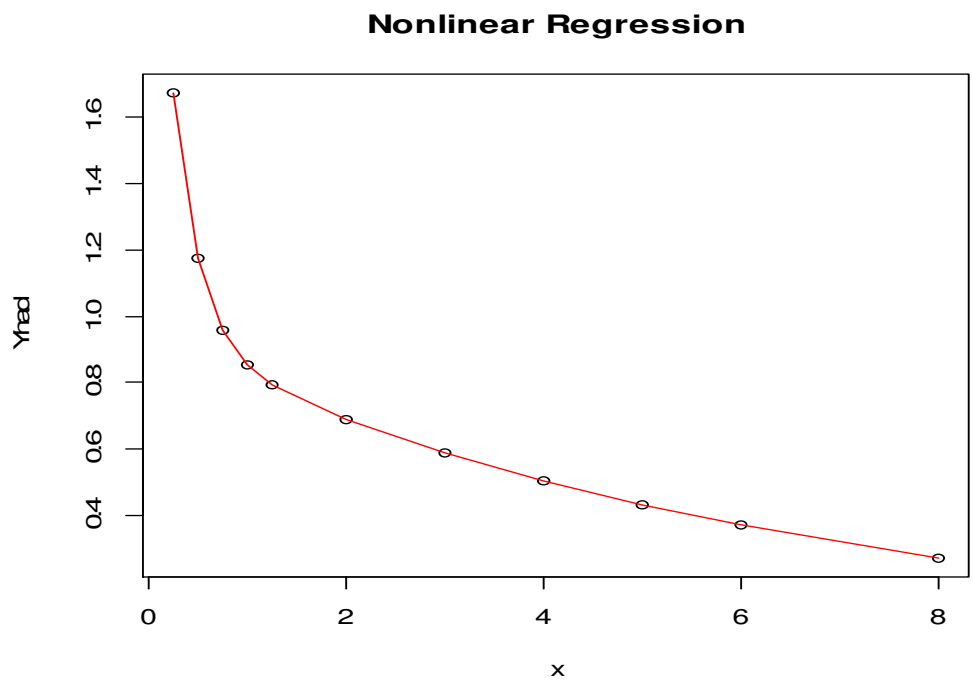

Figure (1): Biexponential Nonlinear Regression Model.

Table (6): The value of Mean Parameters Estimation for Biexponential Robust Nonlinear regression Model by using parameters mean as an initial value from Biexponential Nonlinear Regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.9173893	1.3376692	0.9413825	-1.8725682
	1.9129706	1.3380924	0.9446653	-1.8669093
	1.9120252	1.3379816	0.9451059	-1.8687686

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Table (7): The value of Best Parameters for Biexponential Robust Nonlinear regression Model by using parameters mean as an initial value from Biexponential Nonlinear Regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.999734	1.337423	0.9071101	-1.770363
	1.900491	1.350350	0.9789866	-1.788555
	1.907797	1.350823	0.9683418	-1.825621

Table (8): The values of Akaike Information Criterion and Bayesian Information Criterion for Biexponential robust Nonlinear Regression Model by using parameters mean as an initial value from Biexponential Nonlinear Regression Model.

Sample size	AIC	BIC
	-130.9635985	-112.879743
	-227.280365	-205.730774
	-356.559176	-331.543849

Table (9): The Parameters Test for Biexponential Robust Nonlinear Regression Model by depending on the parameters mean as an initial value from Biexponential Nonlinear Regression Model.

Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
25	φ_1	1.999734	0.38241	5.229	3.41e-07 **
	φ_2	1.337423	0.22847	5.854	1.38e-08 **
	φ_3	0.9071101	0.07135	12.714	< 2e-16 **
	φ_4	-1.770363	0.13236	-13.376	< 2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
50	φ_1	1.900491	0.29051	6.542	1.40e-10 **
	φ_2	1.350350	0.17948	7.524	2.21e-13 **
	φ_3	0.9789866	0.05240	18.683	< 2e-16 **
	φ_4	-1.788555	0.09125	-19.600	< 2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
100	φ_1	1.907797	0.21854	8.73	<2e-16 **
	φ_2	1.350823	0.13300	10.16	<2e-16 **
	φ_3	0.9683418	0.03823	25.33	<2e-16 **
	φ_4	-1.825621	0.06969	-26.20	<2e-16 **

Table (10): The ANOVA Table for Biexponential Robust Nonlinear Regression Model by depending on the parameters mean as an initial value from Biexponential Nonlinear Regression Model.

Sample Size	S.O.V	D.f	S.S	M.S	F-test
25	Regression	4	41.75762	10.439405	239.4255
	Residual Error	271	11.81611	0.043601881	
	Uncorrected Total	275	53.57373		
Sample Size	S.O.V	D.f	S.S	M.S	F-test
50	Regression	4	83.62147	20.9053675	436.9951
	Residual Error	546	26.12004	0.047838901	
	Uncorrected Total	550	109.74151		
Sample Size	S.O.V	D.f	S.S	M.S	F-test
100	Regression	4	162.4692	40.6173	821.6509
	Residual Error	1096	54.17941	0.049433768	
	Uncorrected Total	1100	216.64861		

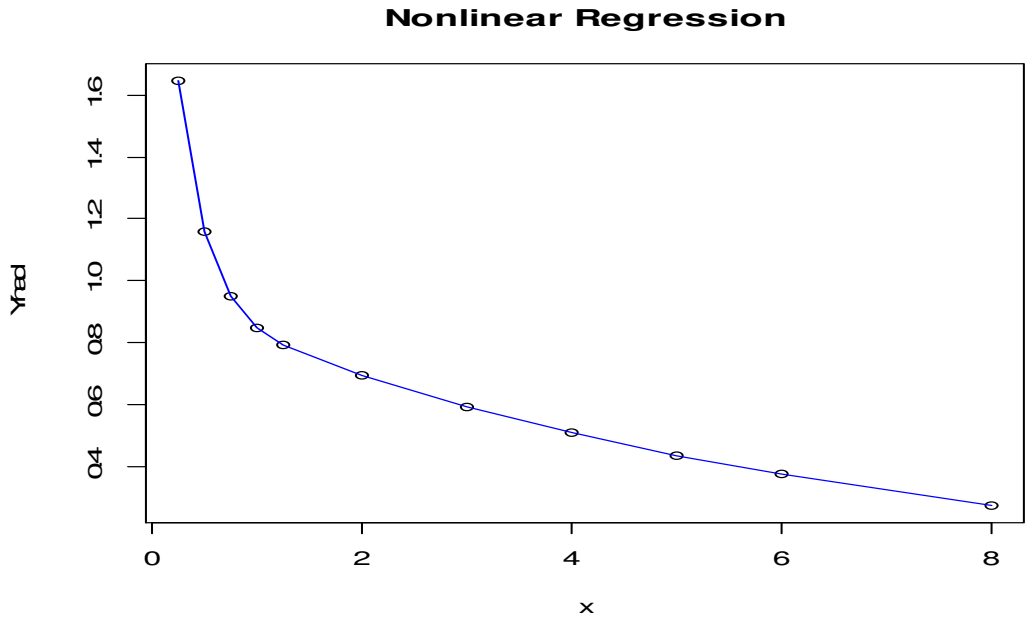

Figure (2): Biexponential Robust Nonlinear Regression Model by using the Parameters Mean.

Table (11): The value of Mean Parameters Estimation for Biexponential Robust Nonlinear Regression Model by using the best parameters as an initial value from Biexponential Nonlinear Regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.9173906	1.3376698	0.9413825	-1.8725680
	1.9129674	1.3380923	0.9446664	-1.8669085
	1.9120141	1.3379819	0.9451097	-1.8687659

Table (12): The value of Best Parameters for Biexponential Robust Nonlinear Regression Model by using the best parameters as an initial value from Biexponential Nonlinear Regression Model.

Sample size	φ_1	φ_2	φ_3	φ_4
	1.999734	1.337423	0.9071100	-1.770363
	1.900475	1.350340	0.9789849	-1.788558
$n_3 = 100$	1.907790	1.350829	0.9683468	-1.825623

Table (13): The values of Akaike Information Criterion and Bayesian Information Criterion for Biexponential Robust Nonlinear Regression Model by depending on the best parameters as an initial value from Biexponential Nonlinear Regression Model.

Sample size	AIC	BIC
	-130.9635993	-112.8797438
	-227.279446	-205.729855
	-356.547958	-331.5326302

Table (14): The Parameters Test for Biexponential Robust Nonlinear Regression Model by depending on the best parameters as an initial value from Biexponential Nonlinear Regression Model.

Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
25	φ_1	1.999734	0.38241	5.229	3.41e-07 **
	φ_2	1.337423	0.22847	5.854	1.38e-08 **
	φ_3	0.9071100	0.07135	12.714	< 2e-16 **
	φ_4	-1.770363	0.13236	-13.376	< 2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
50	φ_1	1.900475	0.29051	6.542	1.40e-10 **
	φ_2	1.350340	0.17948	7.524	2.21e-13 **
	φ_3	0.9789849	0.05240	18.683	< 2e-16 **
	φ_4	-1.788558	0.09126	-19.599	< 2e-16 **
Sample Sizes	Parameters	Estimate	Std. Error	t value	Pr (>|t|)
100	φ_1	1.907790	0.21855	8.729	<2e-16 **
	φ_2	1.350829	0.13301	10.156	<2e-16 **
	φ_3	0.9683468	0.03823	25.332	<2e-16 **
	φ_4	-1.825623	0.06969	-26.195	<2e-16 **

Table (15): The ANOVA Table for Biexponential Robust Nonlinear Regression Model by depending on the best parameters as an initial value from Biexponential Nonlinear Regression Model.

Sample Size	S.O.V	D.f	S.S	M.S	F-test
25	Regression	4	41.75762	10.439405	239.4255
	Residual Error	271	11.81611	0.043601881	
	Uncorrected Total	275	53.57373		
Sample Size	**S.O.V**	**D.f**	**S.S**	**M.S**	**F-test**
50	Regression	4	83.62146	20.905365	436.995
	Residual Error	546	26.12004	0.047838901	
	Uncorrected Total	550	109.7415		
Sample Size	**S.O.V**	**D.f**	**S.S**	**M.S**	**F-test**
100	Regression	4	162.4683	40.617075	821.6459
	Residual Error	1096	54.17944	0.049433795	
	Uncorrected Total	1100	216.64774		

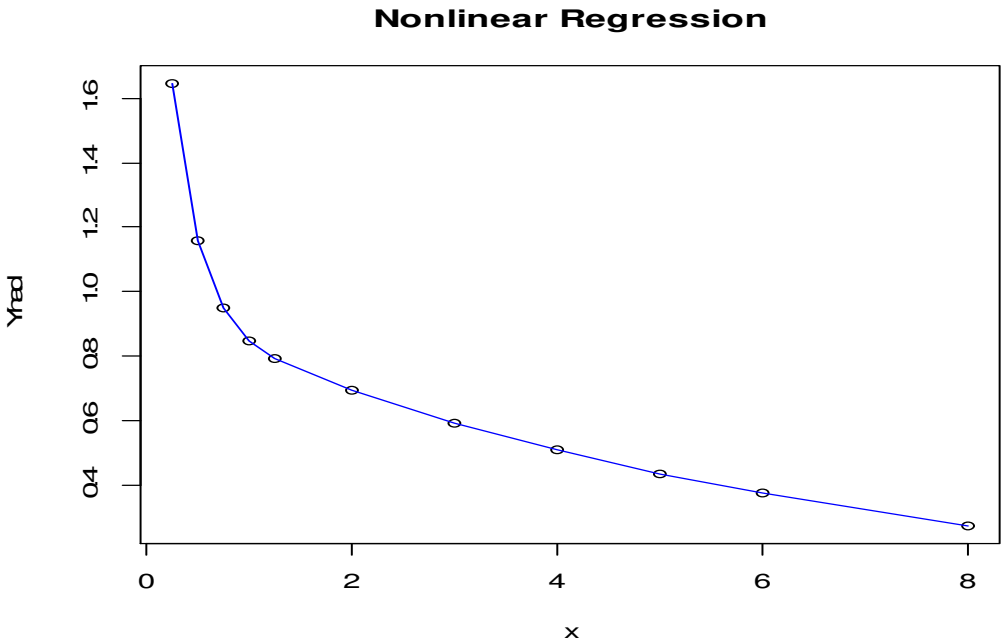

Figure (3): Biexponential Robust Nonlinear Regression Model by using the Best Parameters.

Table (16): The value of Best Parameters in Robust by using M-estimate (Huber and Biweight) Functions.

Sample size	M-estimate (Huber)		M-estimate (Bisquare (Biweight))	
	φ_1	φ_2	φ_1	φ_2
$n_1 = 25$	1.1467938	-0.1347307	1.1466908	-0.1347983
	1.1416988	-0.1352859	1.1421745	-0.135303
$n_3 = 100$	1.1735276	-0.140319	1.1772377	-0.140988

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Table (17): The value of Mean Parameters Estimation in Robust by using M-estimate (Huber and Biweight) Functions.

Sample size	M-estimate (Huber)		M-estimate (Bisquare (Biweight))	
	φ_1	φ_2	φ_1	φ_2
	1.142367872	-0.135815329	1.143630863	-0.136041553
	1.145244716	-0.136366809	1.146289633	-0.136560498
$n_3 = 100$	1.146013212	-0.136387891	1.147087269	-0.136592265

Table (18): The values of Akaike Information Criterion and Bayesian Information Criterion in Robust by using M-estimate (Huber and Biweight) Functions.

Sample size	M-estimate Huber		M-estimate (Bisquare (Biweight))	
	AIC	BIC	AIC	BIC
	62.06881	72.91912	62.05119	72.90151
	176.8746	189.8043	176.925	189.8548
$n_3 = 100$	349.7569	364.7661	350.0742	365.0834

Table (19): The Parameters Test for Robust by using M-estimate (Huber and Biweight) Functions.

Sample Sizes	Functions	Parameters	Estimate	Std. Error	t value
25	Huber	φ_1	1.1467938	0.0247	46.522
		φ_2	-0.1347307	0.0065	-20.6775
	Bisquare	φ_1	1.1466908	0.0254	45.2011
		φ_2	-0.1347983	0.0067	-20.1023
Sample Sizes	Functions	Parameters	Estimate	Std. Error	t value
50	Huber	φ_1	1.1416988	0.0190	59.9761
		φ_2	-0.1352859	0.0050	-26.8867
	Bisquare	φ_1	1.1421745	0.0192	59.5711
		φ_2	-0.135303	0.0051	-26.6973
Sample Sizes	Functions	Parameters	Estimate	Std. Error	t value
100	Huber	φ_1	1.1735276	0.0130	90.4588
		φ_2	-0.140319	0.0034	-40.9196
	Bisquare	φ_1	1.1772377	0.0134	88.1258
		φ_2	-0.140988	0.0035	-39.9281

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Table (20): The ANOVA Table for Robust by using M-estimate (Huber and Biweight) Functions.

Sample Size	Functions	S.O.V	D.f	S.S	M.S	F-test
25	Huber	Regression	2	29.87993	14.939965	206.5889
		Residual Error	273	19.74264	0.072317362	
		Uncorrected Total	275	49.62257		
	Bisquare	S.O.V	D.f	S.S	M.S	F-test
		Regression	2	29.90846	14.95423	206.7994
		Residual Error	273	19.74137	0.07231271	
		Uncorrected Total	275	49.64983		
Sample Size	Functions	S.O.V	D.f	S.S	M.S	F-test
50	Huber	Regression	2	60.24854	30.12427	375.7350
		Residual Error	548	43.93548	0.080174233	
		Uncorrected Total	550	104.18402		
	Bisquare	S.O.V	D.f	S.S	M.S	F-test
		Regression	2	60.26777	30.133885	375.8205
		Residual Error	548	43.93951	0.080181587	
		Uncorrected Total	550	104.20728		
Sample Size	Functions	S.O.V	D.f	S.S	M.S	F-test
100	Huber	Regression	2	129.5914	64.7957	808.1844
		Residual Error	1098	88.03149	0.080174398	
		Uncorrected Total	1100	217.62289		
	Bisquare	S.O.V	D.f	S.S	M.S	F-test
		Regression	2	130.8579	65.42895	815.8473
		Residual Error	1098	88.0569	0.08019754	
		Uncorrected Total	1100	218.9148		

Table (21): Comparison Table.

Biexponential Nonlinear Regression Model				
Sample size	**AIC**		**BIC**	
	-75.7905398		-57.7066843	
	-128.5648466		-107.0152552	
	-227.396284		-202.380957	
Biexponential Robust Nonlinear Regression Model by depending on the parameters mean as an initial value from Biexponential Nonlinear Regression Model				
Sample size	**AIC**		**BIC**	
	-130.9635985		-112.879743	
	-227.280365		-205.730774	
	-356.559176		-331.543849	
Biexponential Robust Nonlinear Regression Model by depending on the best parameters as an initial value from Biexponential Nonlinear Regression Model				
Sample size	**AIC**		**BIC**	
	-130.9635993		-112.8797438	
	-227.279446		-205.729855	
	-356.547958		-331.5326302	
Robust by using M-estimate (Huber and Biweight) Functions				
Sample size	**M-estimate Huber**		**M-estimate (Bisquare(Biweight))**	
	AIC	**BIC**	**AIC**	**BIC**
	62.06881	72.91912	62.05119	72.90151
	176.8746	189.8043	176.925	189.8548
	349.7569	364.7661	350.0742	365.0834

7. Conclusions

Through the application of methods to estimate the parameters and the model, we can deduce the following:

1- Generating data for different sample sizes (**25,50,100**) with (**200**) Reputations for each sample sizes for parameters estimation led to the same Akaike and Bayesian information criterion, except in some cases, where the minimum AIC and BIC were regarded in this paper.

2- In Biexponential Nonlinear Regression Model we estimate four parameters and we found the parameters mean and the best parameters for the model. We tested the parameters they were significant then we found the ANOVA table where the F-test is also significant and we found the Akaike and Bayesian information criterion. To choose the best model we depend on the minimum AIC and BIC.

3- A cording to Biexponential Robust Nonlinear Regression Model**:**

A- By depending on the parameters mean as an initial value from Biexponential Nonlinear Regression Model:

We estimate four parameters. Then we found the mean parameters estimation and the best parameters for the model. The parameters have been tested and the ANOVA table also found they were both significant and we found the Akaike information criterion and Bayesian information criterion. To choose the best model we depend on the minimum AIC and BIC.

B- By depending on the best parameters as an initial value from Biexponential Nonlinear Regression Model:

We estimate four parameters. Then we found the mean parameters estimation and the best parameters for the model. The parameters have been tested and the ANOVA table also found they

were both significant and we found the Akaike information criterion and Bayesian information criterion. To choose the best model we depend on the minimum AIC and BIC.
4- In Robust M-Estimate by using two weight function (Huber and Biweight) we estimate two parameters then we found the mean parameters estimation and the best parameters for the model. The parameters have been tested and the ANOVA table also found they were both significant and we found the Akaike information criterion and Bayesian information criterion. To choose the best model we depend on the minimum AIC and BIC.
5- After comparing all the four practical part with each other for all sample sizes (25,50,100) we can say that the best model is Biexponential Robust Nonlinear Regression Model **by depending on the parameters mean** as an initial value from Biexponential Nonlinear Regression Model for sample size (100) because it has the minimum AIC and BIC.
6- In classical nonlinear regression and also in robust nonlinear regression, we conclude that the sample size affects the results of the estimated value, whenever the sample size increases, the case results of the methods of estimation stay stable more.

8. Recommendations

We recommend:
1- To use computer programs for simulating data and also for actual data.
2- We recommend using all type of robust methods because robust can be use briefly, it can outstretch both of nonlinear regression and non-normality.
3- Of the M-method, there are ten functions of the weights and researchers in statistics can use these functions and compare them with each other.

References

Adler, J. (2009). R in a Nutshell, (Ed.), O'Reilly Media, Inc., 1005 Gravenstein Highway North, Sebastopol, CA 95472, Publication.

Bates, D. M. & Watts, D. G. (1988). Nonlinear regression analysis and its applications, © 1988 by John Wiley & Sons, Inc., Publication.

Fox, J. & Weisberg, S. (2010), Nonlinear Regression and Nonlinear Least Squares in R: An Appendix to An R Companion to Applied Regression, (2nd ed.).

Huber, P. J. & Ronchetti, E. M. (2009), Robust Statistics, (2nd ed.), John Wiley & Sons, Inc., Hoboken, New Jersey, Publication.

Omer, A. W. (2010), Fitting Non-Linear Regression Models to Thalassaemia patients, (Master's Thesis), Collage of Administration and Economics University of Sulaimani.

Pena, D. & Yohai, V. (1999), A Fast Procedure for Outlier Diagnostics in Large Regression Problems, J.A.S.A., Vo1.94, No.446.

Pinheiro, J. C. & Bates, D. M. (2000), Statistics and Computing: Mixed-Effects Models in S and S-Pluss, © 2000 Springer-Verlag New York, Inc.

Ritz, C. & Streibig, J. C. (2008), Nonlinear Regression with R, © Springer Science+Business Media, LLC 2008.

Ronchetti, Elvezio M., (2006), The Historical Development of Robust Statistics, University of Geneva, Switzerland, ICOTS-7, 2006:Ronchetti, Elvezio.Ronchetti@metri.unige.ch.

Salih, S. M. (2011), Comparison among Some Estimation Methods in Generalized Linear Mixed Models by Simulation and Practical Data, (Doctoral dissertation), Collage of Administration and Economics, University of Sulaimani.

SEBER, G. A. F. & WILD, C. J. (2003), Nonlinear Regression, © 2003 by John Wiley & Sons, Inc., Hoboken, New Jersey, Publication.

West, B. T., Welch, K. B. & Gałecki, A. T. (2015), Linear Mixed Models a Practical Guide Using Statistical Software, (2nd ed.), © 2015 by Taylor & Francis Group, LLC.